A Transparent Window Dial

Thibaud Taudin-Chabot

My search for sundials without easily damageable parts was not very successful. So I started thinking and experimenting. Sitting in my chair I tried to visualize the trace the sun would leave in the sky. If I could only get that trace on the window! That was to be the principle of my sundial: the reading point had to be stationary and 'look' at the sky. So all I had to do was construct a sundial which I would place vertically on the glass; the traveling sun would cast its shadow over the stationary point, and during the whole day the shadow pattern would move over this reading point.

Many hours of calculating followed and finally I could draw my sundial. A very familiar pattern emerged on my drawing board which made me think again, for what I saw was a normal vertical sundial for that spot. Had I made a mistake?

No. The proof is very simple. Normally you have the sun, shadow point and sundial pattern in this linear sequence. However, this line remains exactly the same in space if you interchange the latter two of these points to give the sequence: sun, sundial pattern and the 'shadow' point. This shadow point even remains stationary, so you can use it as the reading point - and there it is!

So the recipe is:

- Construct a vertical sundial as if you were mounting it to the exterior of your window.

- Rotate the sundial 180 degrees around the horizontal axis perpendicular to the window. The dial is now laterally inverted and upside down.

- Now mirror the shadow point to the inside of the window around the horizontal axis in the sundial surface. This is the third 180 degree turn made. All is now ready.

I constructed this sundial using an overhead transparency and a colored felt tip pen. When you put figures on the dial, you have to think of constructing a normal vertical sundial on the outside of the window. After following the recipe described above, everything will be OK. Even your text will be perfectly readable as shadow text which moves over your reading point.

It is even possible to create your reading point on the floor and to construct the sundial as if the whole wall is a large window. If that wall actually happens to have only three windows, you use those parts of this large sundial (*i.e.* wall), and paint the appropriate line segments on each window. A scale drawing will be of great help in this case.

Enjoy this new type of sundial. If you construct one following this description, please send me a picture of the result:

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A Stained Glass Diptych Pattern

Fred Sawyer

My immediate reaction upon reading the prior article by member Thibaud Taudin-Chabot was the realization that his basic idea could be applied to produce a stylized stained-glass sundial design!

Consider the web of hour and declination lines characteristic of a vertical dial declining 45°. These two curves are connected at various points by hour lines emanating from a single distant point and providing the internal structure of the webbing.

Suppose now that only the afternoon portion of this dial is used; invert it as Taudin-Chabot suggests. Now combine it with the morning portion of a verticaldial declining 45° to the southeast and similarly inverted.

These two dials join along their noon lines and form a diptych (two-paneled) structure with panels at right angles to each other (each 45° east or west from the meridian plane). The winter solstice and hour lines form a rising sunburst with rays streaming up into the cathedral-like spire formed by the summer solstice line. The single focal point for both panels of the dial sits exactly midway between the panels. The design is perfect for the stained glass artisan!

Equations

In order to draw this pattern for your own latitude, use the equations given here for the afternoon pattern and recall that the morning is just its mirror image. (The equations are intended for Northern hemisphere use; they may require some adjustment in the Southern hemisphere to assure that all angles fall into the correct quadrant.)

Begin by plotting points (x,y coordinates) for the afternoon hours on the summer solstice, when the solar declination is 23.45º. Similarly, now plot points for the afternoon hours on the winter solstice, when the solar declination is -23.45º. Connect the two points for the same hour with a straight line, limiting the line where necessary so that it does not extend below the x-axis.

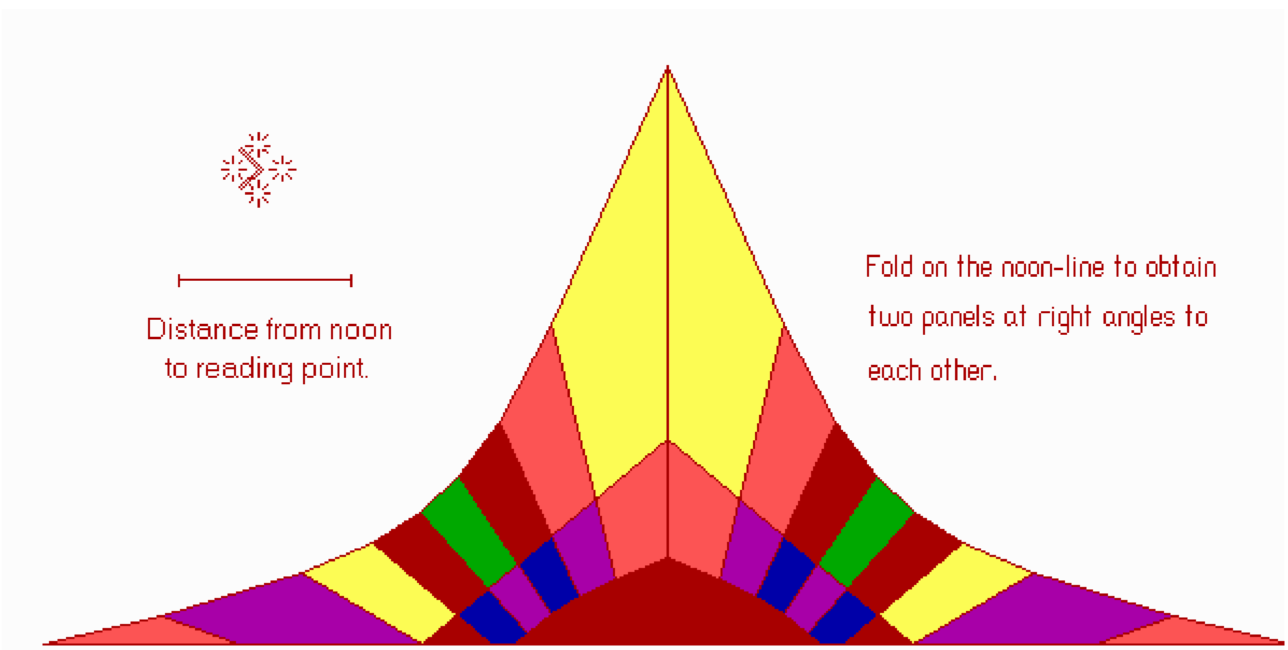
Now draw lines to connect the successive points for the summer solstice; and continue with a similar set of lines for the winter solstice points.

A similar procedure may be followed for an equinox line, which, unlike the solstice lines, will be one straight line from noon to its intersection with the x-axis.

Mirror the pattern around the noon line so that a similar panel is drawn for the morning hours. The resulting pattern must be folded along the noon-line to form two panels which stand at a 90º angle to each other. This diptych is then placed standing in the sun with its angle opening to the north and with the meridian (true north-south line) running down the middle of the angle, bisecting it.

As the final touch, a single point on the meridian line and between the two panels must be highlighted. This point's distance from the noon-line junction of the two panels is 1.414214 (based on whatever unit measure was adopted for the drawing of the hour-lines).

As the sun moves around the south of the diptych and its light shines through the stained glass, a colored pattern of light flows across the horizontal plane to the north of the panels. The hour-lines, which mark the change from one color to another, sweep across this highlighted point just to the north of the panels, and the correct solar time is registered here by their crossing.



φ = Latitude
δ= Solar Declination for the Given Day
t = Solar Hour-Angle Past Noon (15°/hr.)

$$L = \sin^{-1}\left(-0.5\sqrt{2}\cos\varphi\right)$$
$$T = \sin^{-1}\left(0.5\sqrt{2}/\cos L\right)$$
$$T = \cot^{-1}\left(\sin\varphi\right)$$
$$A = \tan^{-1}\left(0.5\sqrt{2}/\tan\varphi\right)$$

$$q = \cos L\cos\left(T-t\right) + \sin L\tan\delta$$
$$m = \frac{\sin(T-t)}{q}$$
$$n = \frac{\sin L\cos(T-t) - \cos L\tan\delta}{q}$$

$$x = 1 - m\cos A - n\sin A$$
$$y = m\sin A - n\cos A$$

Distance from noon to reading point = $\sqrt{2}$

Solving the Spherical Triangle

Fred Sawyer

I had mixed feelings when my daughter began studying trigonometry in high school. It certainly is true that the formulas of gnomonics are replete with the basic trig functions, so any modern approach to dialing should have such a course as a prerequisite. However, in reviewing her textbook I found that the course stopped short of dealing with the spherical trigonometry which provides the modern rationale for most of the gnomonic formulas.

A trip to the library established that none of the newer texts on the shelves included anything but a cursory mention of spherical trigonometry. The older texts, closer to the 19th than to the 21st century, provided more depth in their treatment but reworked all the formulas and presented them in formats designed for evaluation by log tables or slide rules (lamentable casualties of the electronic age).

My goal in this article is to present enough basic information on spherical trigonometry to permit the reader (familiar with plane trig) to solve any spherical triangle and to derive many of the essential equations of spherical astronomy and gnomonics.

Consider a sphere, and draw an arc on its surface. If the arc is part of a circle whose center is also the center of the sphere itself, then the arc is part of a great circle. For example, on the earth, any piece of the equator or a line of longitude is a great circle arc, since the center of the circle from which it comes is the center of the earth. However, any non-equatorial latitude line is part of a minor circle which does not share a center with the globe and is therefore not a great circle arc.

A spherical triangle is a triangle on the surface of a sphere, each of whose sides is an arc (less than 180 degrees) of a great circle. Any three non-collinear points on the sphere's surface define a triangle.

Lengths of the sides of the triangle are given in angular measure; each side's length equals the measure (between 0° and 180°) of the angle formed by its end points with the sphere's center as vertex.

Spherical triangles satisfy the following five rules:

1. The sum of the lengths of a spherical triangle's sides is always less than 360°.

2. The sum of the angles at its vertices is greater than 180° and less than 540°.

3. The sum of the lengths of any two sides is greater than the length of the third side.

4. If a side (or angle) differs from 90° by more than another side (or angle) on a spherical triangle, then it is