Sundials With Punctiform Hour Lines

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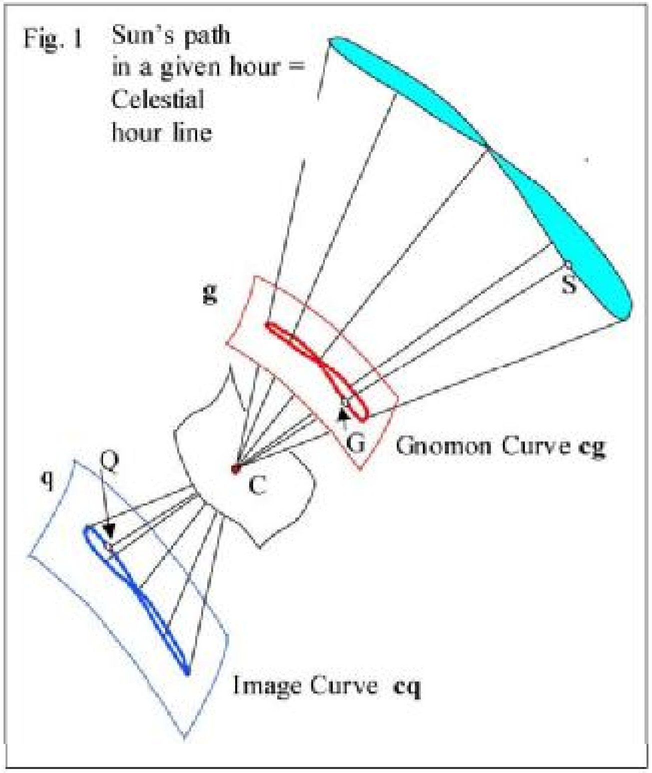


Part 1 – General Description

Introduction

Let us consider the line drawn in the sky by the points occupied by the Sun's center at a given hour on different days of the year (celestial hour line), a point **C** in space and two surfaces **g** and **q,** of any shape. Let us suppose that the two surfaces are placed in such a way that a ray from the Sun intersects first the surface **g** (gnomonic surface) and then the surface **q** (dial or image surface).

If we project from the point **C** the points of the celestial hour line, the straight projecting lines (rays) cross the two surfaces and draw on them two curves with similar shapes, **cg** and **cq**.

If we suppose finally that the surface **g** is transparent and **q** opaque, and if we draw the two curves **cg** (gnomon curve) and **cq** (image curve) obtained with the projection, we have a sundial.

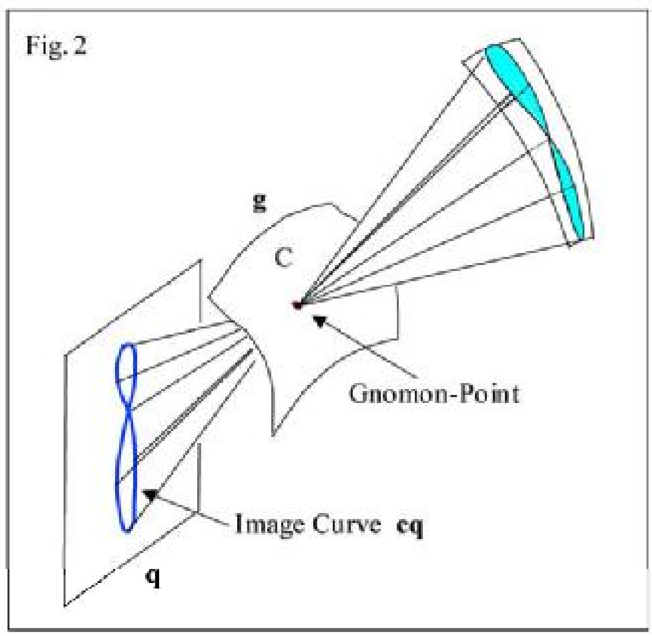
On any day of the year when the Sun S lights the surface **g** and the point G, it generates a ray of shadow that, passing through the center of projection C, hits the surface **q** in the point Q.

If the center of projection C doesn't belong to either of the two surfaces **g** and **q,** then the sundial that we obtain is very difficult to read and of no practical utility. The situation is different when the center C belongs to one of the two surfaces.

Let us examine the possible cases.

Center of projection on the gnomonic surface g (Fig. 2)

The gnomon curve **cg** is reduced to the single point C and the sundial becomes a usual sundial, but with a punctiform gnomon. If the gnomonic surface **g** is transparent and the point C is opaque, the image surface is reached by a shadow-ray, as in usual sundials. If instead **g** is



opaque and the point C is transparent (a hole) the image surface **q** is illuminated by a ray from the Sun and we have a dark-room sundial (or a pin-hole sundial).

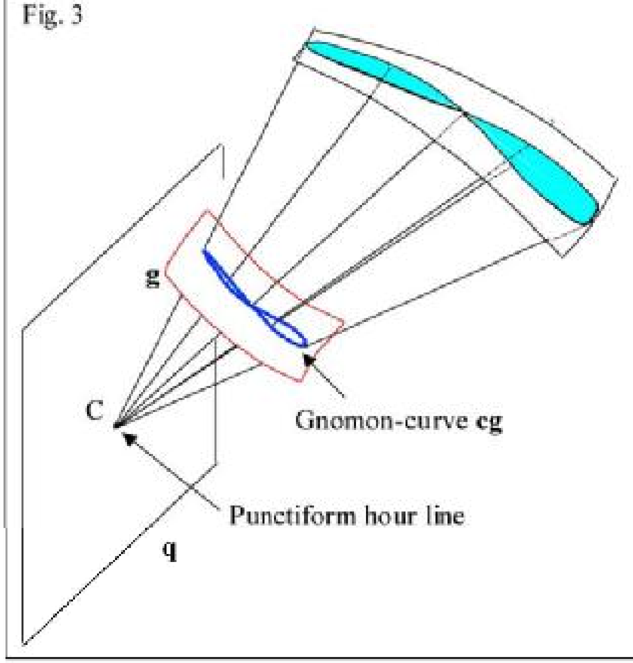
Center of projection on the image surface q. Sundials with one punctiform hour line. (Fig. 3)

The hour line on the sundial (surface **q**) is reduced to the single point C and we have therefore a sundial with one punctiform hour line.

On the surface **g** we need a gnomon having the shape of the curve **cg**.

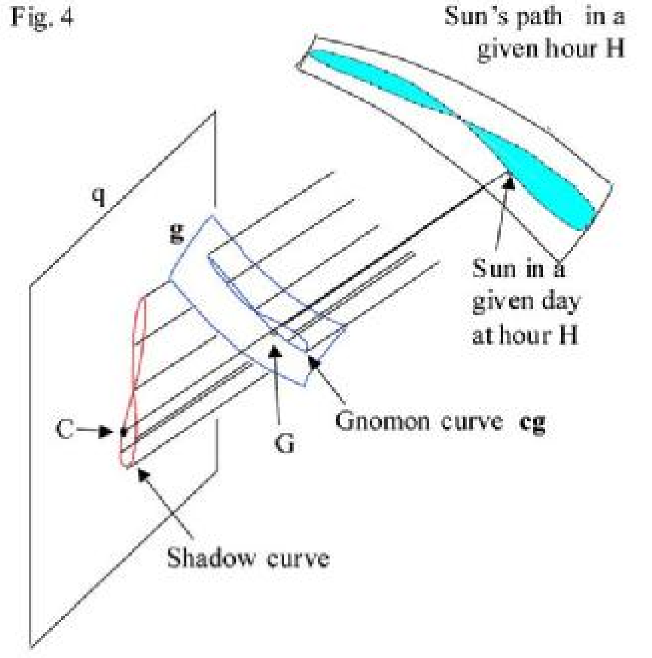
Whenever the Sun's parallel rays illuminate the gnomon curve and project it onto the image surface, the result is a curve of shadow.

Fig. 3



Unless the celestial hour line is an analemma, this shadow curve will pass over the point C only at the instant corresponding to the considered hour. (Fig. 4)

Fig. 4

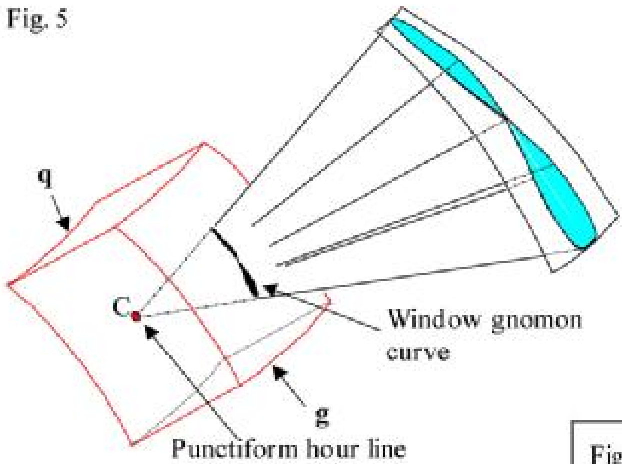


We may have different cases:
- transparent gnomonic surface **g** with opaque gnomonic curve (linear gnomon); (Fig. 3)
- transparent gnomonic surface **g** with opaque zone contained within the gnomonic curve (sheet gnomon);
- opaque gnomonic surface **g** with transparent zone contained within in the gnomonic curve, (fissure-gnomon-curve or window-gnomon-curve) (Fig. 5)

In the last case the zone between the window-curve and the punctiform hour line C can be "solid", that is, formed by a real masonry in which there is a funnel-shaped hole that goes from the point C to the window-curve (fig. 5), or formed by two surfaces, separated by a cavity with closed sides, on which there is a hole for the point C and an appropriately cut window-curve.

In part 2 of this article I will describe this type of sundial in greater detail.

Fig. 5



We may also build a reflection sundial with a punctiform hour line. While in the common reflection sundials the mirror is "punctiform" (that is, of small dimensions) and stands in for the gnomonic point, now it is the whole gnomonic curve that has to be entirely reflecting. An example is shown in fig. 6.

In the preceding discussion I have supposed, for simplicity, that the gnomon curve belongs to a surface – however, this hypothesis is not necessary. In fact it is enough that the gnomon curve has its points on the radii that go from the points of the celestial hour line crossed by the Sun to the projection center C.

One can then plans gnomon curves with all kinds of spatial forms or that bound a part of a two or three-dimensional structure. At the given hour the shadow of this curve will fall on the punctiform hour line (v. fig. 7)

Fig. 6

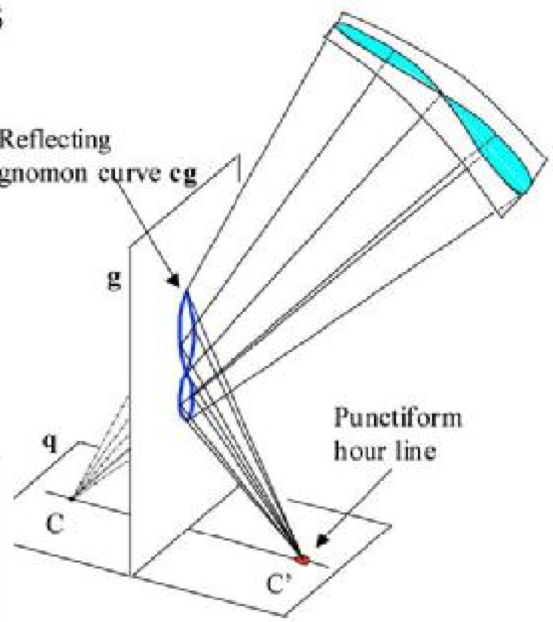
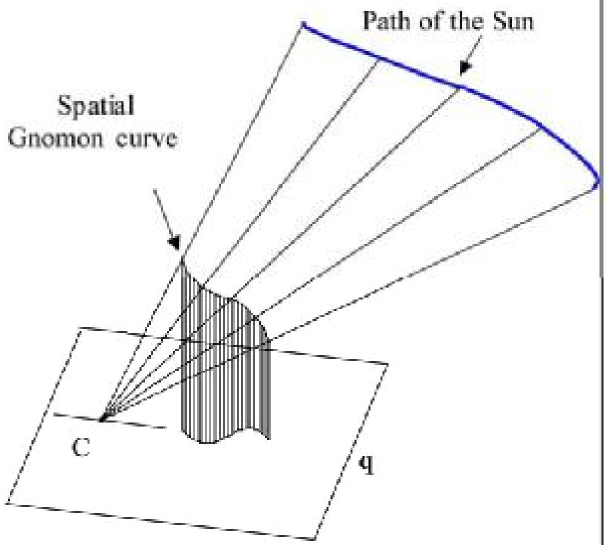


Fig. 7



In all the figures I have drawn the celestial hour line with the shape of an analemma, that is I have supposed a line of mean time. Obviously a sundial with a punctiform hour line can have hour lines with True or Mean Time, with modern or ancient hours (Italic, Babylonian, etc.) and can be built for an any hour of the day, not limited to noon.

It is possible also to build sundials that show more hours, but in that case we have rather to speak of gnomonic complex.

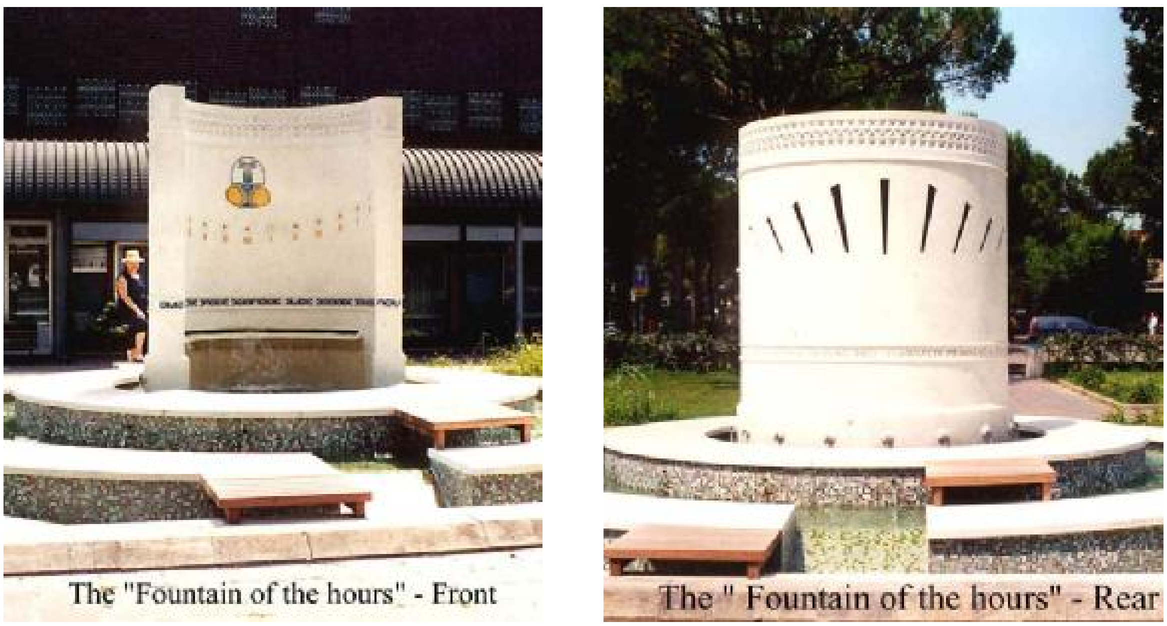
In fact we obtain a sundial with more hours building more sundials, each with one punctiform hour line, or by using a single center of projection C for all the different dials, or, for easier reading and comprehension, putting close together some independent dials, having calculated each one for a different hour.

Some sundials with movable gnomons lying within the the hour planes can be considered with a punctiform hour line.

Part 2 – Sundial With Hour Windows

Description and characteristics

The sundial called “Fontana delle ore” ("Fountain of the hours") shown in Figs. 8 and 9, designed and constructed in 2001 in the park of the thermal baths of Cervia (Ravenna – Italy) by the well-known Italian dialist Mario Arnaldi, is an example of a "sundial with hour windows or with punctiform hour lines" built on a three-dimensional structure delimited by two surfaces separated by a considerable thickness.



The "Fountain of the hours" - Front

The " Fountain of the hours" - Rear

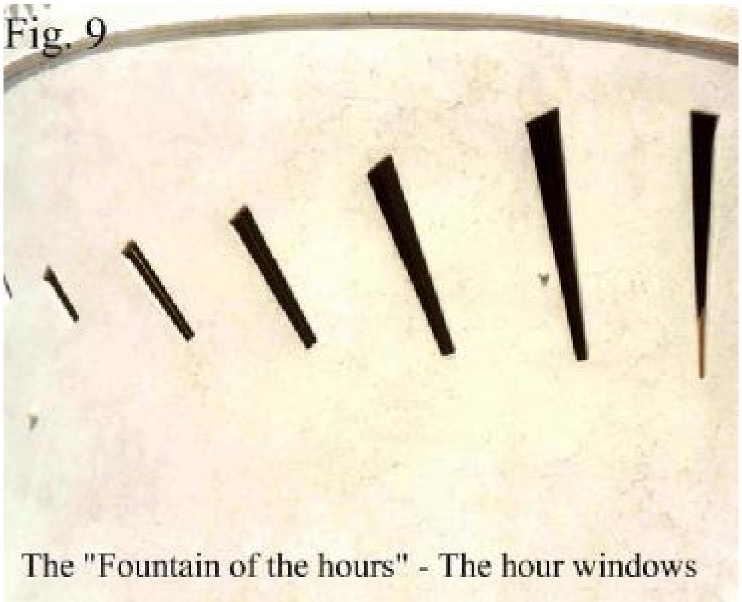
Figure 8.

In sundials of this type the two surfaces, one illuminated by the rays of the Sun and the other partially or totally in shadow, may have any shape and may be separated by a thickness which is not necessarily uniform.

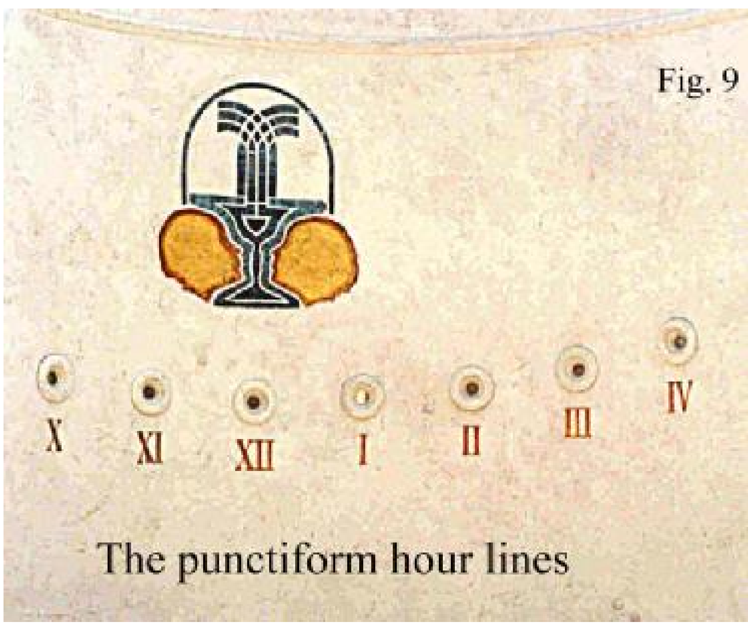
Since every little window, on the surface in shade, can be set in the position that the designer prefers and chooses for aesthetical, practical or personal reasons, the structure should not be considered a simple sundial, but rather a gnomonic complex formed by as many sundials as there are hour windows.

Each one of these single dials is a sundial with only one hour line reduced to a point. In the "Fountain of the hours" the two surfaces are coaxial vertical cylinders separated by a constant thickness.

Obviously we may build sundials based on the same principle with different structures.



The "Fountain of the hours" - The hour windows



The punctiform hour lines

For instance using:

- a wall or a vertical plate placed in the East - West plane
- a reclining wall arranged as a roof
- a horizontal plate put at a given height (i.e. on a platform roof)
- a part of a spherical surface
- parts of cone or truncated cone
- vertical pyramids with many sides (i.e. one for every hour)
- concave or convex surfaces of any shape.

How it works

Let us consider a structure bounded by two surfaces separated by a given thickness, one exposed to the Sun's rays and the other in shade. (Fig. 8)

Let us place on the shaded surface a set of points W_H, one for every wanted hour. I will call each of these points "point of the hour H" or "window of the hour H".

In "The Fountain" these points are little circular windows with a diameter of 2 cm, covered with colored glass (Fig. 9)

The ray of the Sun that in the instant H would arrive on the window W_H, if the wall were transparent, hits the external surface of the same wall at a point whose position changes on the different days of the year. Joining these points we obtain a curve that I will call "curve of the hour H."

If we perforate the wall with a set of holes from the points of the hour H curve to the window W_H, or we cut the wall with a funnel-shaped "fissure" that gives the same result, we get a sundial in which the window of the hour is illuminated, only in the instant H, on any day of the year. (Fig. 10)

If the structure is empty, that is if the two surfaces are as the walls of a box, the window and its corresponding hour curve can be made as cuts on the two surfaces.

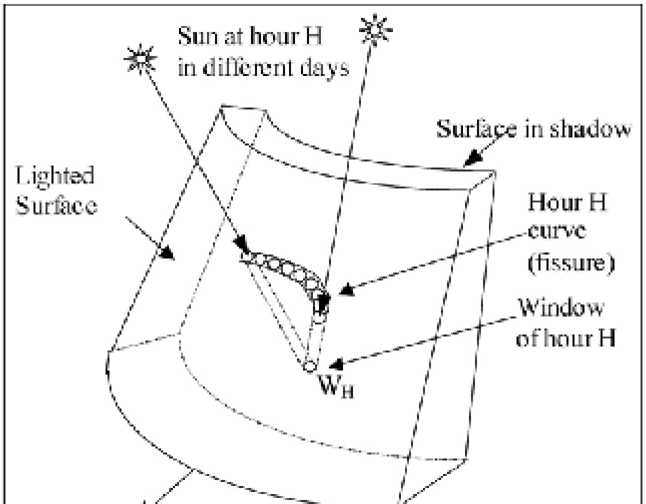


Fig. 10

In Fig. 9 we see the "cuts" in the shaded surface of "The Fountain". These "fissures" have a width calculated so that the sunlight illuminates the corresponding inside window for about 8 to 10 minutes before and after the considered hour.

Analytical description

Given an orthogonal coordinates system Oxyz, with the xy plane parallel to the horizontal plane and with the axis z vertical toward the Zenith. Then,

$$S(x, y, z) = 0 \quad (1)$$

the equation of the surface on which we will make the "fissures";

$F_H(x_H, y_H, z_H)$ the point on the shaded surface, center of the window of the hour H; $\{m_1, m_2, m_3\}$ the direction cosines that characterize, in the coordinate system chosen, the direction of the center of the Sun's disk at the instant H of a given day of the year.

A generic point P belonging to the ray passing through the point W_H, that is, belonging to the ray that hits the window of the hour H, has its coordinates given by:

$$x = x_H + k \cdot m_1$$
$$y = y_H + k \cdot m_2 \qquad (2)$$
$$z = z_H + k \cdot m_3$$

where k is a parameter (k = 0 for P coincident with W_H)

Replacing the (2) in (1) we have the equation :

$S(x_H + k \cdot m_1, y_H + k \cdot m_2, z_H + k \cdot m_3) = 0$ with *k* the only unknown.

Finally, replacing in (2) the obtained value of k, we have the coordinates of the point of the fissure relative to the given day.

For structures with a vertical axis of symmetry (cylinders, cones, polyhedrons, pyramids, etc.) it is advisable to take the window of the hour H at a point in which the shaded surface is cut from the vertical plane passing through the axis of the structure with an azimuth equal to that of the Sun in the days of the Equinoxes at the time H.

A Particular case - Vertical Cylindrical Surfaces (Fig. 11)

System of coordinates with:

- x axis in the North-South direction
- y axis in the East-West direction
- z axis, vertical, positive toward the Zenith

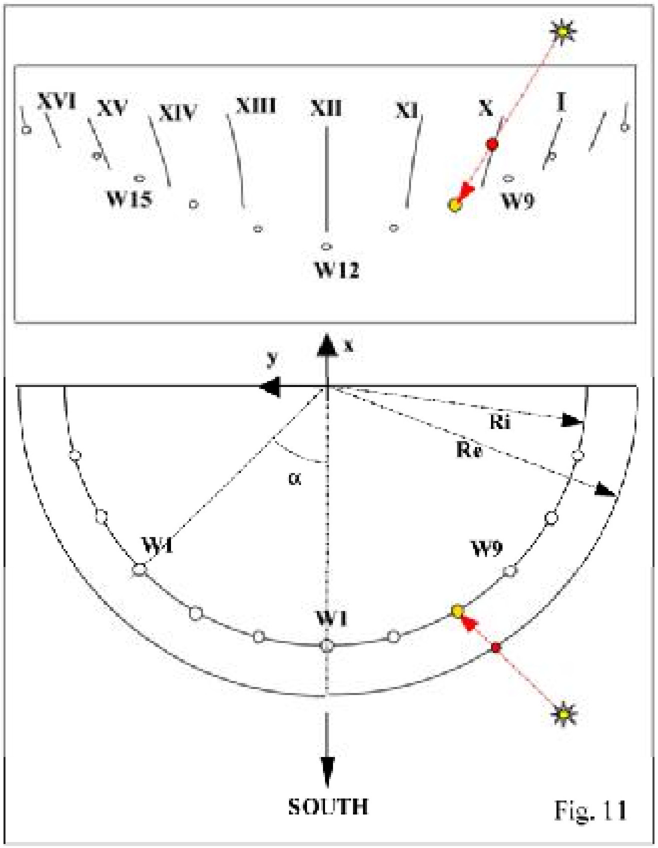


Fig. 11

The two cylindrical surfaces have their axes coincident with z and radius Ri (shaded surface internal) and Re (illuminated surface, external). The equation of the external surface is

$$S(x, y, z) = x^2 + y^2 - \mathrm{Re}^2 = 0 \quad (3)$$

If we set:

- α the angle between the North-South direction and the window of the hour H, that is, the azimuth of W_H;
- Az the Sun's azimuth in the day of calculation, at the hour H;
- h the Sun's height in the same instant;

we obtain the following formulas:

$$m_1 = \cos(h) \cdot \cos(Az)$$
$$x_H = -\mathrm{Re} \cdot \cos(\alpha)$$
$$m_2 = -\cos(h) \cdot \sin(Az)$$
$$y_H = +\mathrm{Re} \cdot \sin(\alpha)$$
$$m3 = -\sin(h)$$
$$z_H = 0$$

Replacing these expressions in the Eqs. (2) and then in the expression (3) we find the equation

$$k^2 + 2 \cdot k \cdot \frac{m_1 \cdot x_H + m_2 \cdot y \cdot}{1 - {m_3}^2} - \frac{(\mathrm{Re}^2 - Ri^2)}{1 - {m_3}^2} = 0$$ from which we obtain the value of k.

Finally with the equations (2) we have the coordinates of the point at which the Sun's ray, in the given day at hour H, hits the external surface before coming to the corresponding inside window.

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